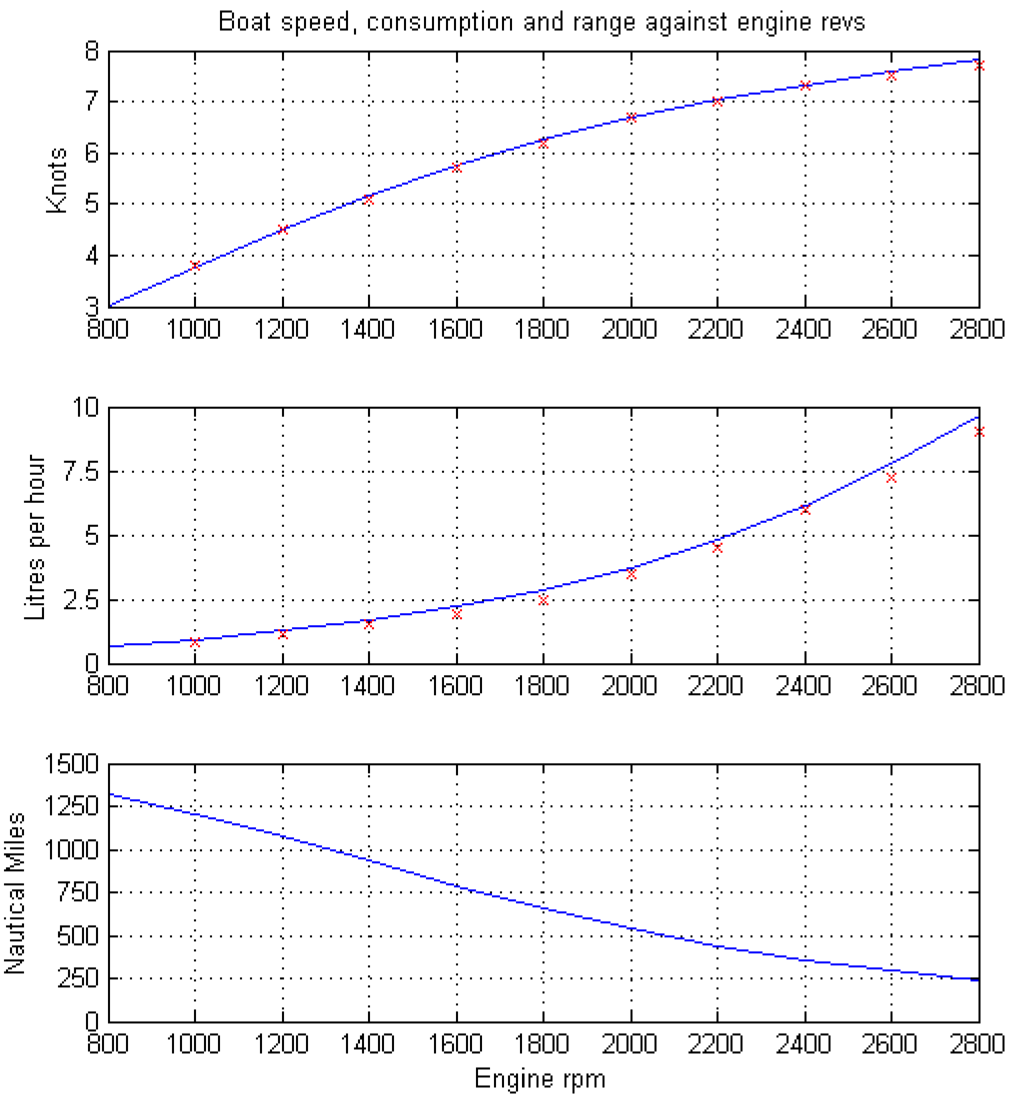
Analysis of the fuel consumption of a Rustler 42 as a function of boat and engine speed

Summary

This started is an analysis of the range I could expect from my Rustler 42 which has a 50HP Beta engine and a Gori 3-blade folding prop. To calculate this I took the following steps:

1. Calculate the water resistance (aka drag) of the hull as a function of boat speed
2. Add effect of wind to get total drag
3. Calculate the thrust from the prop as a function of boat speed and prop rpm
4. Equate thrust from the prop (from 3) with drag (from 2) to get speed as a function of engine rpm
5. Calculate the torque seen at the prop-shaft, again as a function of rpm
6. Multiply the torque by rpm to get shaft kW or HP
7. Convert kW to fuel consumption per hour
8. And finally get range as a function of engine rpm and thus also of speed

I made some measurements to validate the results. In the graphs simulations are shown as solid lines and measurements or independent data are shown as red crosses.



It's also possible to re-do the calculations to take account of head winds, adverse tides or waves, which allows one to work out the optimum cruising speed in these cases.

Finally I added a couple of graphs which can be calculated from the foregoing analysis and are possibly interesting: (i) pull in reverse plotted as an equivalent wind velocity and (ii) equivalent wind strength to equal the force exerted by a current or tide. These are potentially useful when anchoring.

The whole exercise was for me to learn more about my R42, but in case useful or interesting to others I post results here. If I've made a gross error I'd be pleased to be told so!

Hull drag

The elements of hull drag are:

1. Friction or viscous drag
2. Wave making drag
3. Wind resistance from the vector addition of the wind and the boat's own motion

The standard Rayleigh equation for resistance: $R = 0.5\rho \cdot U^2 \cdot S \cdot C$ applies to all three but the drag coefficient C is a complicated function, especially for wave drag since it is dependent on both hull shape and on speed. Each is analysed below:

Viscous drag

This is just friction. It dominates at low speed and rises quite steadily with increasing speed at slightly less than speed squared, and is dependent strongly on wetted area. The coefficient depends on Reynolds number as below:

$$C_f = 0.075/(\log_{10}(R_{num}) - 2)^2$$
$$R_{num} = U \cdot L/\nu$$
$$R_v = 0.5\rho \cdot U^2 \cdot S \cdot Cf$$

Wave drag

This is negligible at very low speeds but rises very abruptly as the boat approaches a critical speed, often referred to as 'hull speed'. In practice there is no such thing as hull speed - with more force the boat will always go faster – but forces do rise very steeply.

The coefficient C_w is a function of Froude number and of hull shape. It is usually derived from numerical modeling with CAD programs, but I derived the simplified exponential formula below as an empirical fit to the Michell triple integral (Michell, J H, "The wave resistance of a ship". Philosophical Magazine Vol 45, 1898).

$$C_w = 0.0022 \cdot \exp((F_{num} - 0.33)/0.057)$$
$$F_{num} = U/\sqrt{g \cdot L}$$
$$R_w = 0.5\rho \cdot U^2 \cdot S \cdot Cw$$

where:

C_f = *Coefficient of friction*

C_w = *Coefficient of wave resistance*

R_{num} = *Reynolds Number*

F_{num} = *Froude Number*

U = *Speed*

L = *Waterline Length*

ν = *Kinematic Viscosity*

R_v = *Viscous Drag*

R_w = *Wave Drag*

ρ = *Density of sea water*

g = *aceleration due to gravity*

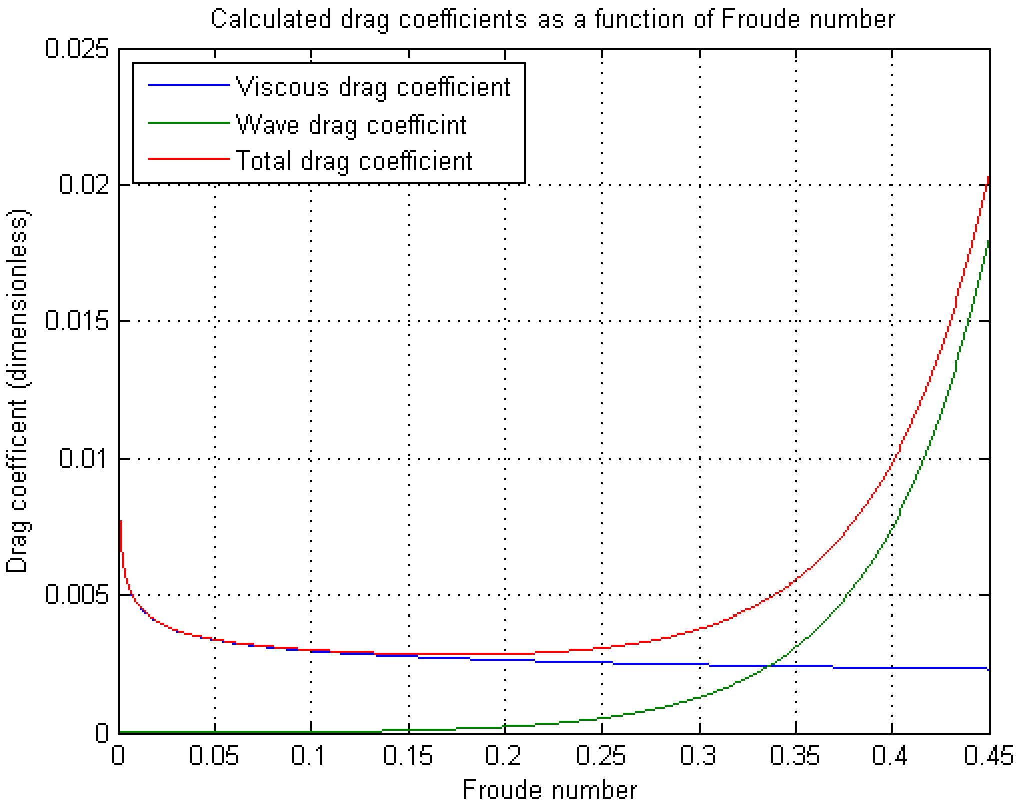
S = *Surface area of hull*

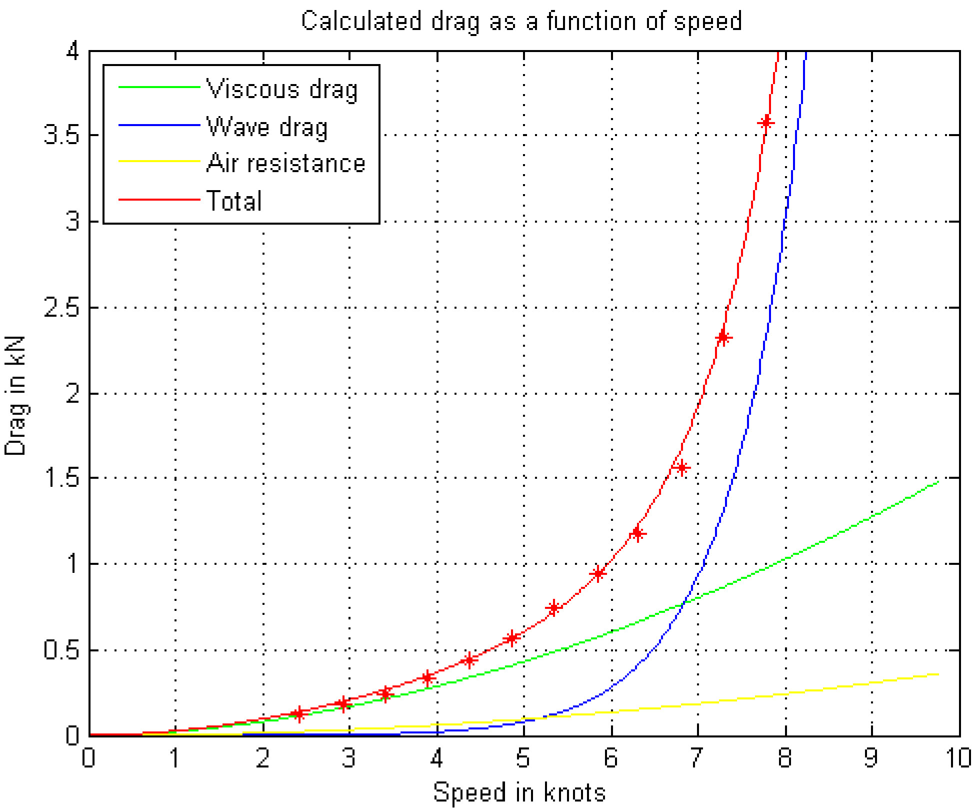
Wind resistance

Rather than measure the cross section of the windage I simply measured the horizontal component of the force on the boat whilst anchored one afternoon. This is not strictly accurate as it ignores the fact that the wind speed is a function of height when due to moving air (aka wind!), but is uniform when the boat is being pushed through still air. Nonetheless since it is only a small component compared to the water drag this error can be ignored.

I measured 640N in a steady 13 knot wind. Using the square law dependence on wind speed, this gives 136N (ie a little below 14kg of force) at 6 knots which compares to about 1000N from the water drag.

Total hull drag



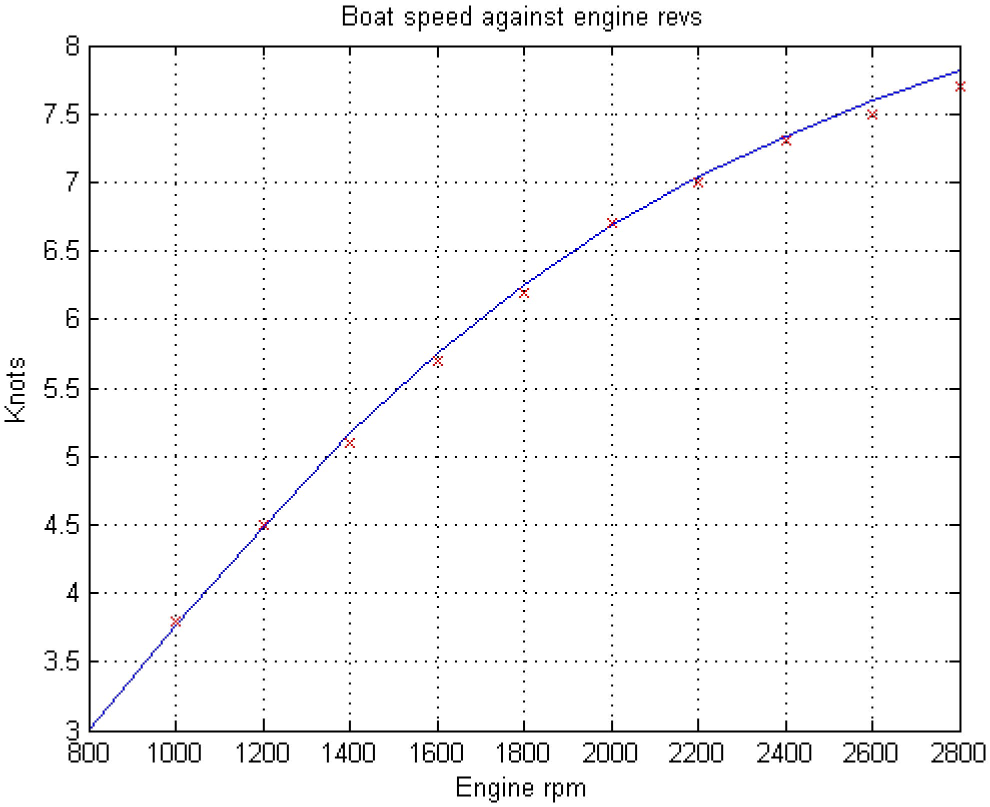


The red curve is the sum of the viscous, wave and air drag and is the force required to push along a Rustler 42 hull in still water. The red crosses are the result of doing the same modeling using the freeware hull design program “HullDrag 32”, see http://www.zwakenberg.de/hulldrag/

The agreement is rather good, which adds some support for the numbers. Note that force is expressed in kilo Newtons: 1kg force is 9.8N, so to drive the hull at 6kts needs about 100kg of force.

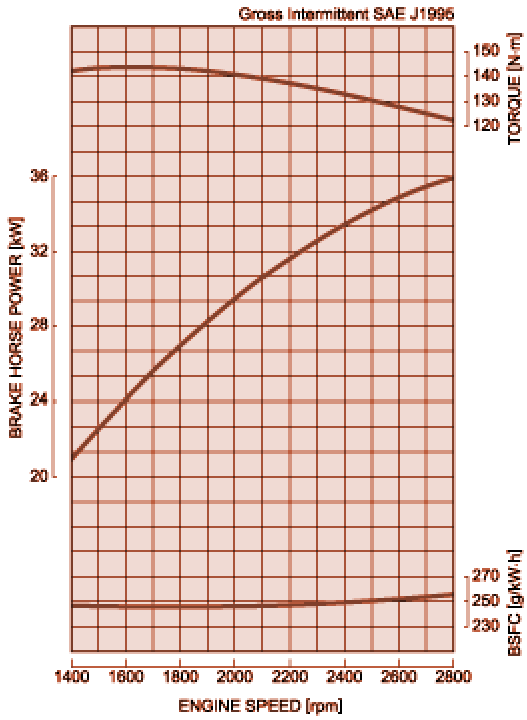
Boat speed as a function of rpm

Adding the propeller size, pitch, efficiency and gear ratio gives the following graph. The red crosses were measured and the blue simulated. The agreement is quite good, especially at lower speeds where I am most interested.



Engine efficiency

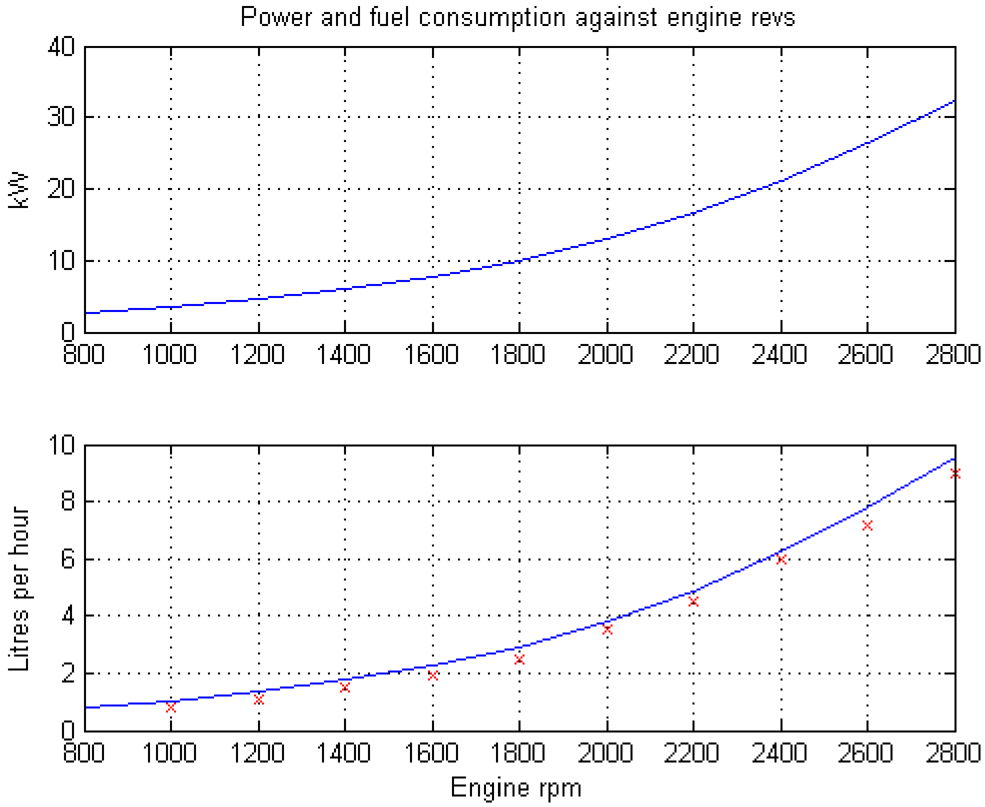
Actually all engines are pretty much the same, whether 5kW or 5MW, consuming 245 to 250 g / kWhr of energy delivered. The density of diesel is 0.83 which gives 0.295 litres per kWhr. The consumption is also almost independent of engine speed, see the BSFC graph for the V2203-M Kubota engine as used by both Nanni and Beta. The optimum speed for best efficiency is very broad, it's pretty much flat anywhere between 1400 and 2200 rpm.



V2203-M Performance Curve

HP and Consumption per hour

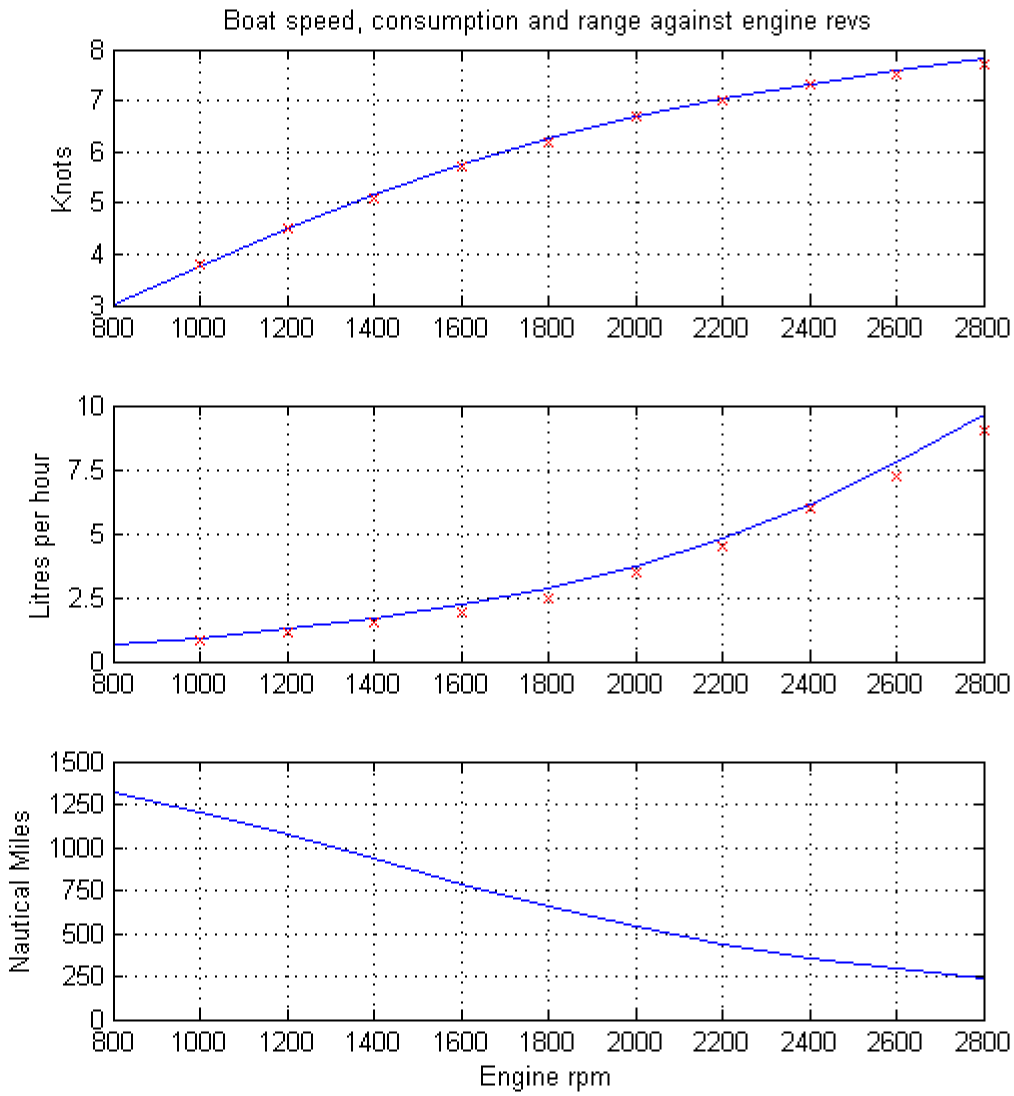
Plotting the used HP in kW, and thus the consumption per hour, gives the composite graphs below:



The red crosses are data taken from the Beta 50 manual. Again the agreement is quite good and actual consumption being very slightly higher in practice agrees with my impression. It also agrees quite well with a rule of thumb that the boat will use 3 litres / hr at 6kts.

Range

Range is given by capacity (300 litres in my boat) divided by the litres per hour, multiplied by the speed. All three parameters are plotted here.

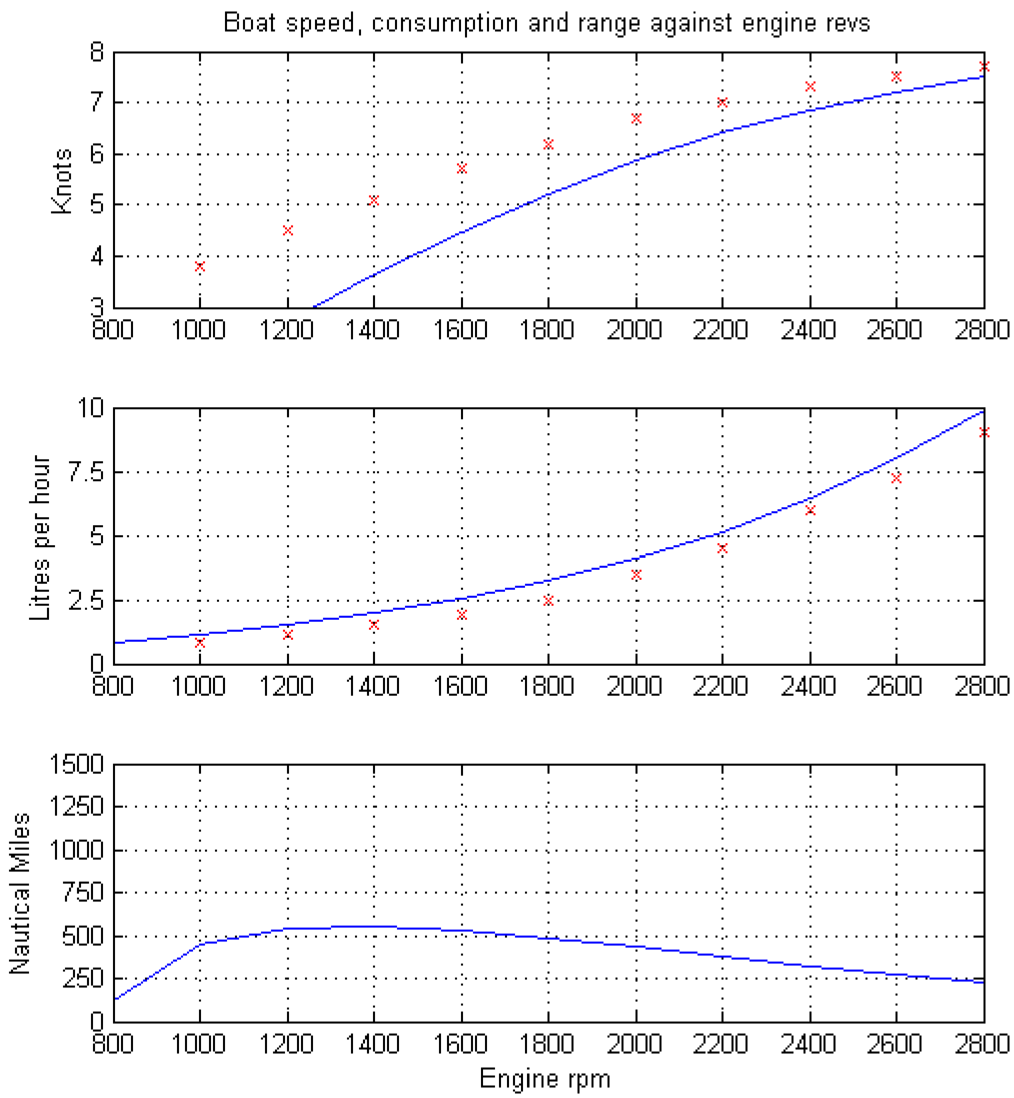


The key point to note is that, in still air and water at least, the *range always increases as one goes slower*.

My original purpose was to see if I could get 1000 miles range without adding an extra tank. It seems I can, provided I go slowly enough and there's no headwind. Since studying this graph I now tend to motor slower than I used to!

Effect of a head-wind

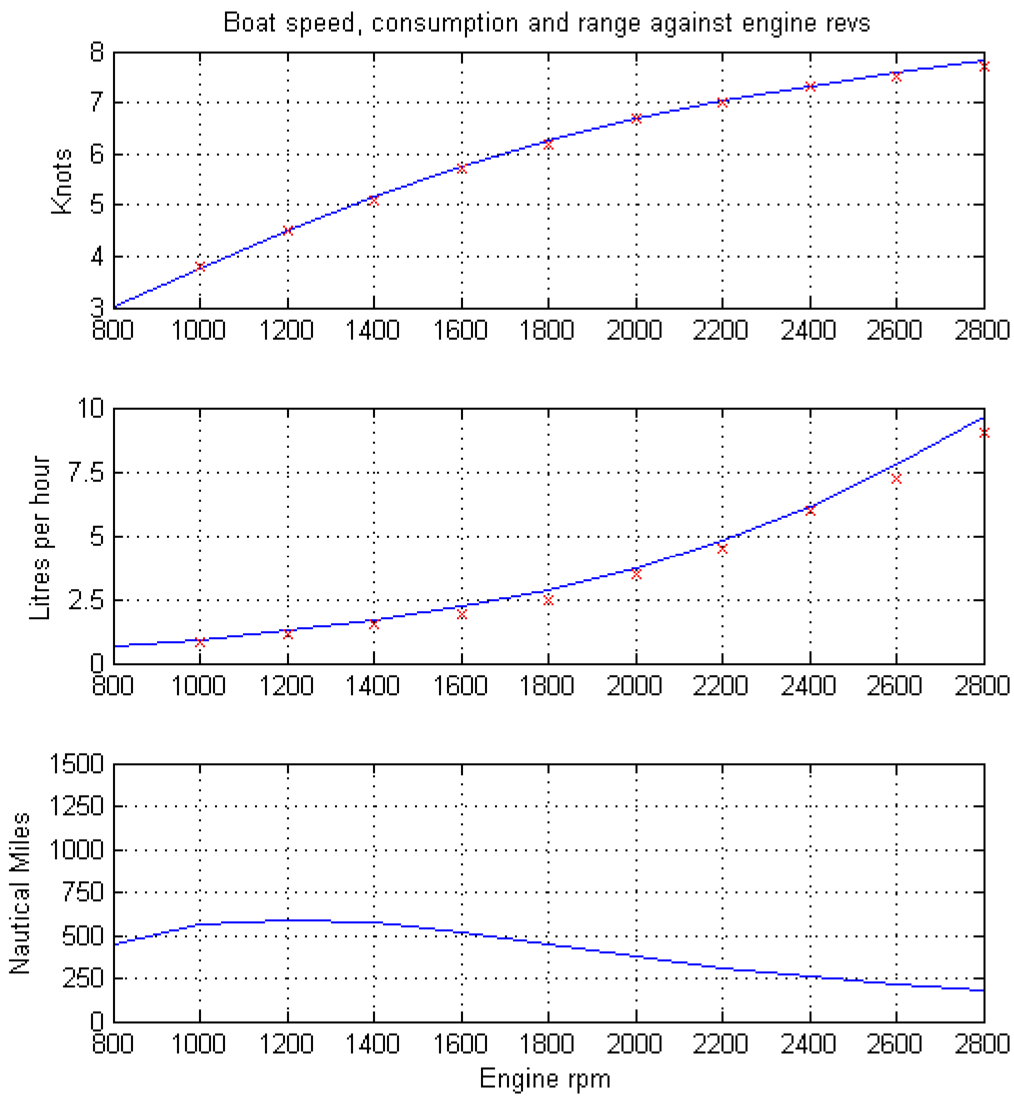
The graph above can be re-plotted with a 10 knot headwind (true headwind, so apparent will be higher).



There is now a clear optimum at around 1400 rpm giving only 550 miles range.

Effect of an adverse tide

The graph can also be plotted for no wind but a 2 knot adverse tide:



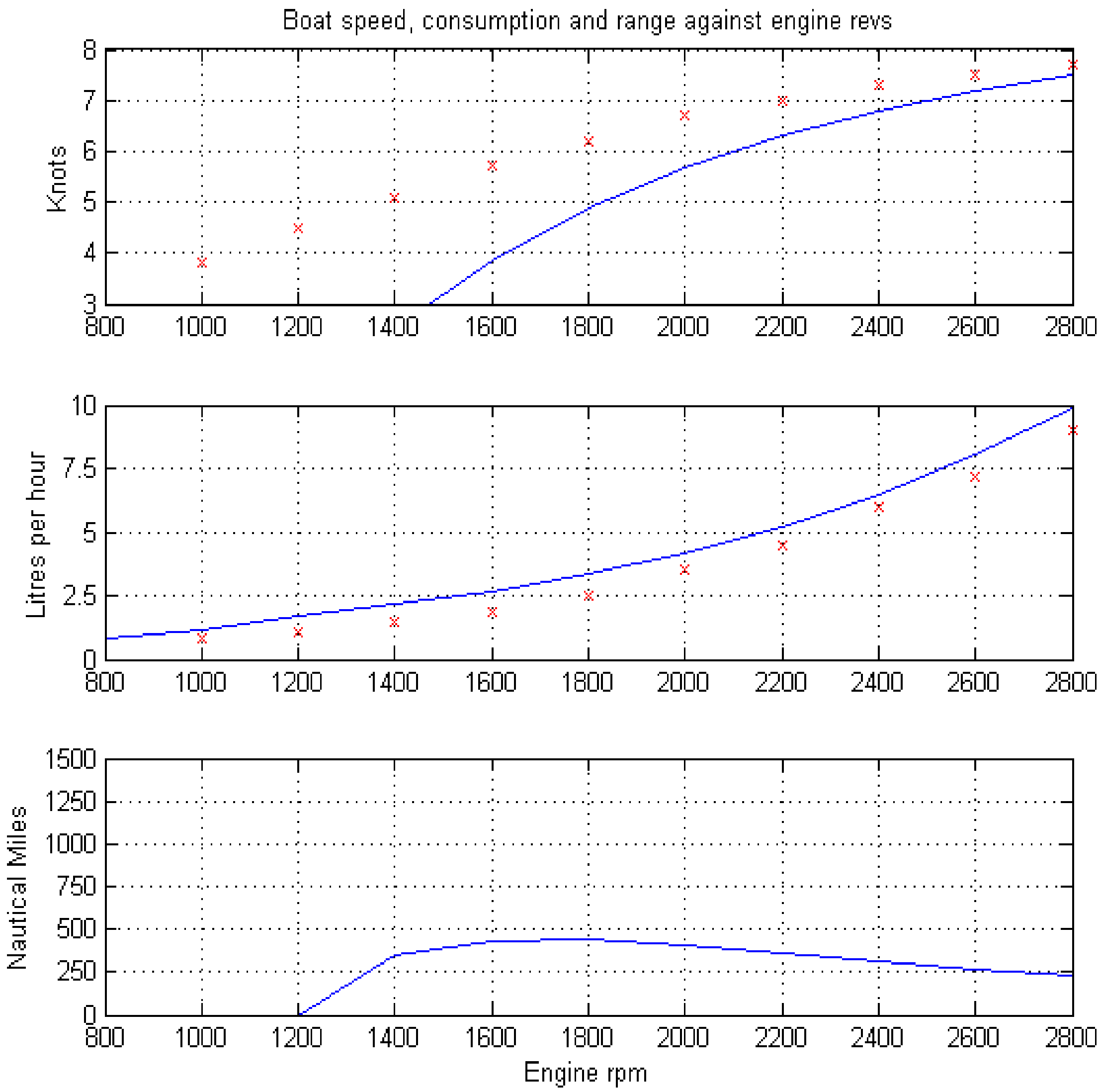
The effect is less dramatic than a headwind, but there is nonetheless an optimum at around 1200 rpm which would give 4.5kts through the water but only 2.5kts SoG. It's a pretty broad maximum and 1600rpm uses only about 16% more fuel but gives 3.7kts SoG, an improvement of nearly 50%.

A quick sanity check shows that if at 1200 rpm you get 4.5kts and 1000 miles range in still water, then with a 2kt adverse tide you'll get around 2.5kts SoG and the same number of engine hours, so will go 1000x 2.5 / 4.5 = 555 miles.

Effect of waves

The force exerted by waves depends on their length and amplitude, so a general treatment is difficult or impossible. However in open sea there is a relationship between length and amplitude, reducing the problem. Pioneering work was done by Professor Gerritsma and others at the University of Delft, and their results for a 10m LWL yacht are used here without further analysis.

The peak resistance occurs at a wave period of around 3.5 seconds. The R42 has a length / displacement ration of around 4.6, so the maximum force due to waves in open water is around 950N. The corresponding wave height of the upper 1/3 of the waves is around 1.1 metres. If the wave height is different, the force will scale as approximately the ratios of wave heights squared, so scaling the result for waves of 1m height gives a wave induced force of $950 / 1.1^2 = 785$N. This is plotted here:

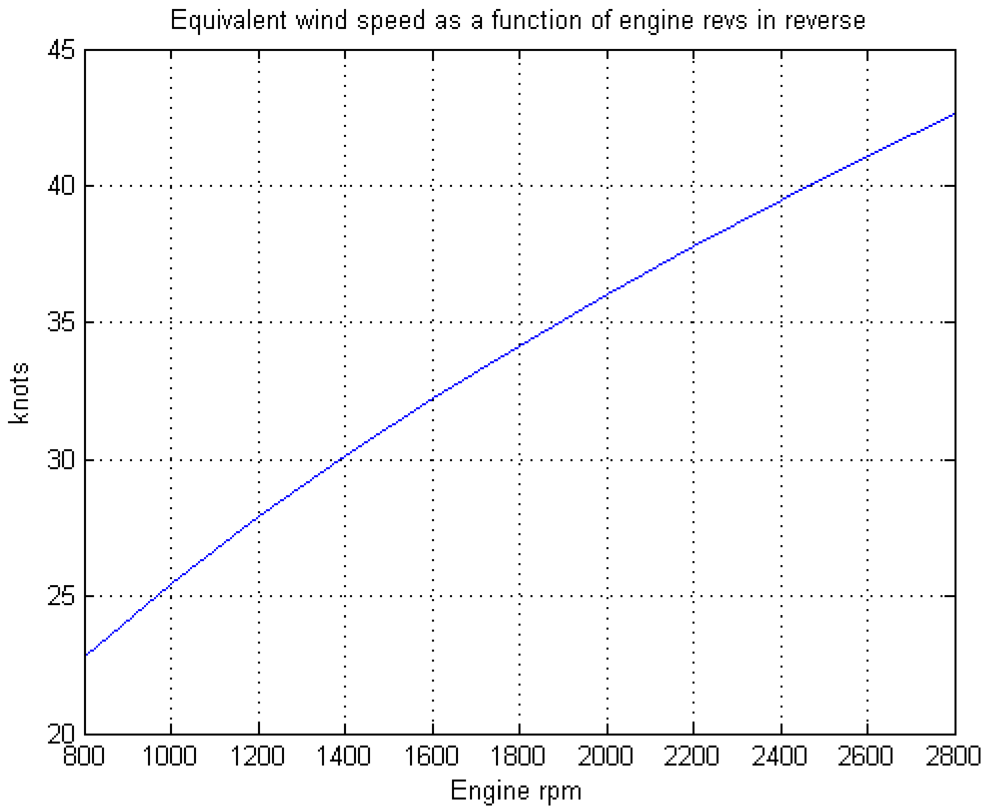


The optimum engine speed is now around 1800rpm, corresponding to a boat speed of 5 knots, and range has fallen to only about 400 miles (from nearly 1000 in still water). A significant wave height of 1.0 metres is perhaps quite large, but nonetheless it indicates that waves have an extremely significant effect.

Equivalent wind to reverse pull

Knowing gearbox ratios, the same method can be used to calculate reverse thrust when the boat is stationary, aka ‘bollard pull. As I have a Gori prop I have assumed the propeller parameters are the same in reverse as they are in forward, but the gearbox ratio is slightly different.

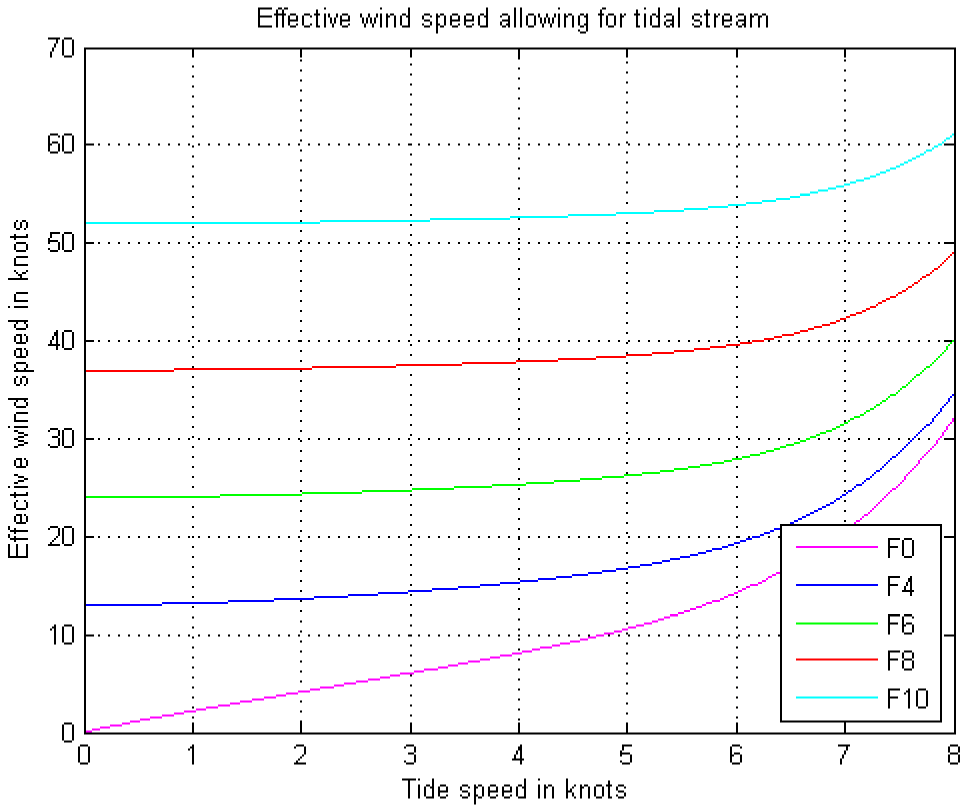
The graph shows the rpm needed to exert the same force on the anchor as would a steady wind. No account has been taken of sheering or other dynamic forces.



Thus to put a pull on the anchor equivalent to a 34knot wind (ie just F8) one should reverse at 1800rpm.

Equivalent wind force to a tidal flow

Since we know force due to wind and force due to water flow, the two can be equated to give an equivalent wind strength increase due to tide through an anchorage. This is plotted here for various wind strengths from calm (F0) to Beaufort 10.



It can be seen that if anchored in a steady 13kts (F4), the effect of a 2kt tide through the anchorage is equivalent to a wind increase to only 14kts or so. Alternatively if there is no wind but only tide, the effect of a 5knot tide is only equivalent to 10kts of wind (F3). This is much less than is usually assumed – or maybe R42 hulls are particularly well designed...